

Homework X

Problem 1 [20 points]

For this problem, use the single sort W with signature;

$a: \square W$

$b: \square W$

$f: W, W, W, \square W$

$g: W, W \square W$

with equations expressed as the following rewrite rules:

for all $x, y \square W$

$f(a, b, x) \square f(x, x, x)$

$g(x, y) \square x$

$g(x, y) \square y$

- (a) Is this rewriting system terminating (i.e., does every rewriting chain starting with a ground term eventually halt)?
- (b) Is this rewriting system confluent? (i.e., are there ground terms that are equivalent using the corresponding equations but lead to different normal forms).

Justify your answers.

Problem 2 [20 points]

For this problem, use the signature of the integers, namely;

$0: \square \text{Int}$

$s: \text{Int} \square \text{Int}$ (intending the successor function)

$-: \text{Int} \square \text{Int}$ (intending the negative of an integer)

$+: \text{Int}, \text{Int} \square \text{Int}$ (intending the sum of two integers)

The rewriting rules below model equations we normally expect.

For all $x, y, z \square \text{Int}$

$s x + y \square s (x+y)$

$0 + x \square x$

$x + 0 \square x$

$(-x) + x \square 0$

$(x+y) + z \square x + (y+z)$

- (a) Is this rewriting system terminating (i.e., does every rewriting chain starting with a ground term eventually halt)?
- (b) Is this rewriting system confluent? (i.e., are there ground terms that are equivalent using just the corresponding equations that lead to different normal forms).

Justify your answers.

Note that for either of these problems, you are welcome, but not obligated, to use Miranda to explore the rewriting outcomes of the systems.