SAMPLE

Exam II
Open book/notes

1. [20 points]
Consider the context-free grammar (start = S) with $\Sigma = \{0,1\}$ and productions
$S \to \varepsilon \mid 1S \mid S0 \mid 0S1$.
What language is generated? Prove your answer.

2. [25 points]
Take $\Sigma = \{0,1\}$. For $x \in \Sigma^*$, let $\neg x$ denote the "one's complement" of x — that is, the string obtained from x by exchanging '0's and '1's (and $\neg\varepsilon = \varepsilon$). For instance, $\neg 0110 = 1001$. Then define $L = \{x \in \Sigma^* \mid \neg x = x^R\}$, where x^R is the reversal of x. For instance, 0101 and 1010 are in L, but 0110 is not. Provide a context-free grammar for L, and justify its correctness.

3. [25 points]
Consider the language $L = a^* b^* - \{a^n b^n \mid n \geq 0\}$. Provide a PDA that recognizes this language. Clearly explain the operation of your PDA (including your choice of empty stack or final state recognition), and justify its correctness.

4. [30 points]
Take $\Sigma = \{a,b\}$, and let $L = \{a^p b^q \mid p \geq 0 \text{ and } q = p*r \text{ for some integer } r\}$. Thus the strings in L have all 'a's preceding all 'b's, and the number of 'b's is a multiple of the number of 'a's; for instance, $a^2 b^6$ and $a^3 b^{18}$ are in L, but $a^2 b^5$ is not. Is L context-free? Prove your answer.