1. 20 [points]
Provide a regular expression describing all sequences (Σ = {0,1}) that do not contain 01 as a subsequence, and justify your answer.

2. [30 points]
Match each NFA with an equivalent regular expression selected from (i) – (iv). No justification is required.

(i) ε + 0 (00 + 01*1)* 01*
(a) 
\[
\begin{array}{ccc}
0 & 1 & 0 \\
0 & 1 & 0 \\
1 & 0 & 1 \\
\end{array}
\]

(ii) ε + 0 (00 + 10*1)* 0
(b) 
\[
\begin{array}{ccc}
0 & 1 & 0 \\
0 & 1 & 0 \\
1 & 0 & 1 \\
\end{array}
\]

(iii) ε + 0 (00 + 01*1)* 0
(c) 
\[
\begin{array}{ccc}
0 & 1 & 0 \\
0 & 1 & 0 \\
1 & 0 & 1 \\
\end{array}
\]

(iv) ε + 0 (10 + 10*1)* 1

3. [30 points]
Determine whether or not \( \{0^p1^q \mid p,q \geq 0 \text{ and } 2p > 3q \} \) is regular and prove your answer.

4. [20 points]
One of the following functions \( f: \{0,1\}^* \to \{0,1\}^* \) is extendible and of finite index (i.e., can be realized with a DGSM) and one is not. Which is which, and why?

(a) for each \( x \in \{0,1\}^* \), \( f(x) \) is \( x \) with all instances of '1' deleted
(b) \( f(\epsilon) = \epsilon \), and for each \( x \in \{0,1\}^* \) and \( \lambda \in \{0,1\} \), \( f(x\lambda) = f(x) x\lambda \),