Example 1 *Using the four-decimal-place computer described in the last example, solve*

$$0.729x_1 + 0.81x_2 + 0.9x_3 = 0.6867$$
$$x_1 + x_2 + x_3 = 0.8338$$
$$1.331x_1 + 1.21x_2 + 1.1x_3 = 1.000$$

Its exact solution, rounded to four places, is

$$x_1 = 0.2245, \qquad x_2 = 0.2814, \qquad x_3 = 0.3279 \tag{6.52}$$

Solution Without Pivoting

$$\left[\begin{array}{ccc|c} 0.7290 & 0.8100 & 0.9000 & 0.6867 \\ 1.000 & 1.000 & 1.000 & 0.8338 \\ 1.331 & 1.210 & 1.100 & 1.000 \end{array}\right]$$
$$\downarrow \quad \begin{array}{l} m_{21} = 1.372 \\ m_{31} = 1.826 \end{array}$$
$$\left[\begin{array}{ccc|c} 0.7290 & 0.8100 & 0.9000 & 0.6867 \\ 0.0 & -0.1110 & -0.2350 & -0.1084 \\ 0.0 & -0.2690 & -0.5430 & -0.2540 \end{array}\right]$$
$$\downarrow \quad m_{32} = 2.423$$
$$\left[\begin{array}{ccc|c} 0.7290 & 0.8100 & 0.9000 & 0.6867 \\ 0.0 & -0.1110 & -0.2350 & -0.1084 \\ 0.0 & 0.0 & 0.02640 & 0.008700 \end{array}\right]$$

The solution is

$$x_1 = 0.2251, \qquad x_2 = 0.2790, \qquad x_3 = 0.3295 \tag{6.53}$$

Solution With Pivoting

To indicate the interchange of rows i and j, we will use the notation $r_i \longleftrightarrow r_j$.

$$\left[\begin{array}{ccc|c} 0.7290 & 0.8100 & 0.9000 & 0.6867 \\ 1.000 & 1.000 & 1.000 & 0.8338 \\ 1.331 & 1.210 & 1.100 & 1.000 \end{array}\right]$$

$$r_1 \longleftrightarrow r_3 \downarrow \begin{array}{l} m_{21} = 0.7513 \\ m_{31} = 0.5477 \end{array}$$

$$\left[\begin{array}{ccc|c} 1.331 & 1.210 & 1.100 & 1.000 \\ 0.0 & 0.09090 & 0.1736 & 0.08250 \\ 0.0 & 0.1473 & 0.2975 & 0.1390 \end{array}\right]$$

$$r_2 \longleftrightarrow r_3 \downarrow m_{32} = 0.6171$$

$$\left[\begin{array}{ccc|c} 1.331 & 1.210 & 1.100 & 1.000 \\ 0.0 & 0.1473 & 0.2975 & 0.1390 \\ 0.0 & 0.0 & -0.01000 & -0.003280 \end{array}\right]$$

The solution is

$$x_1 = 0.2246, \qquad x_2 = 0.2812, \qquad x_3 = 0.3280 \tag{6.54}$$

Comparing this to (6.52), we observe that this is a much more accurate solution than (6.53). □