

Example 6.5.2. Use a computer with four digit floating-point decimal arithmetic with rounding, and use Gaussian elimination with pivoting. The system to be solved is

$$\begin{array}{rccccrcr}
 x_1 & + & & 0.5x_2 & + & 0.3333x_3 & = & 1 \\
 & & 0.5x_1 & + & 0.3333x_2 & + & 0.25x_3 & = & 0 \\
 0.3333x_1 & + & & 0.25x_2 & + & & 0.2x_3 & = & 0
 \end{array} \tag{6.85}$$

Then

$$\begin{aligned}
 x^{(0)} &= [8.968, -35.77, 29.77]^T \\
 r^{(0)} &= [-0.005341, -0.004359, -0.0005344]^T \\
 \hat{e}^{(0)} &= [0.09216, -0.5442, 0.5239]^T \\
 x^{(1)} &= [9.060, -36.31, 30.29] \\
 r^{(1)} &= [-0.0006570, -0.0003770, -0.0001980]^T \\
 \hat{e}^{(2)} &= [0.001707, -0.01300, 0.01241]^T \\
 x^{(2)} &= [9.062, -36.32, 30.30]^T
 \end{aligned}$$

The iterate $x^{(2)}$ is the correctly rounded solution of the system (6.85). This illustrates the usefulness of the residual correction method.