A QUADRATURE EXAMPLE

$$I = \int_O^\pi e^x \cos x \; dx = -\frac{e^\pi + 1}{2} \doteq -12.0703463163896$$

The error and asymptotic error:

$$\begin{aligned} E_n^T &= -\frac{h^2\,(b-a)}{12} f''\,(c_n)\,, \qquad f(x) = e^x \cos x \\ \left|E_n^T\right| &\leq \frac{h^2\pi}{12} \cdot 14.921 = 3.906h^2 \equiv \mathit{Bound} \\ \widetilde{E}_n^T &= -\frac{h^2}{12}\left[f'(\pi) - f'(0)\right] \\ &= \frac{h^2}{12}\left[e^\pi + 1\right] \doteq 2.012h^2 \end{aligned}$$

Using the trapezoidal rule:

n	T_n	$I - T_n$	*Ratio*	*Bound*
2	−17.38925933	5.319		9.638
4	−13.33602285	1.266	4.20	2.409
8	−12.38216243	$3.118E-1$	4.06	$6.024E-1$
16	−12.14800410	$7.766E-2$	4.02	$1.506E-1$
32	−12.08974212	$1.940E-2$	4.00	$3.765E-2$
64	−12.07519410	$4.848E-3$	4.00	$9.412E-3$
128	−12.07155818	$1.212E-3$	4.00	$2.353E-3$
256	−12.07064928	$3.030E-4$	4.00	$5.822E-4$

Using the corrected trapezoidal rule:

n	T_n	$I - T_n$	$\widetilde{E}_n^T$	CT_n	$I - CT_n$	*Ratio*
2	−17.38925933	5.319	4.964	−12.42552836651093	$3.552E-1$	14.4
4	−13.33602285	1.266	1.241	−12.09509010646616	$2.474E-2$	15.6
8	−12.38216243	$3.118E-1$	$3.103E-1$	−12.07192924452925	$1.583E-3$	15.9
16	−12.14800410	$7.766E-2$	$7.757E-2$	−12.07044580359025	$9.949E-5$	16.0
32	−12.08974212	$1.940E-2$	$1.939E-2$	−12.07035254293755	$6.227E-6$	16.0
64	−12.07519410	$4.848E-3$	$4.848E-3$	−12.07034670568297	$3.893E-7$	16.0
128	−12.07155818	$1.212E-3$	$1.212E-3$	−12.07034634072256	$2.433E-8$	16.0
256	−12.07064928	$3.030E-4$	$3.030E-4$	−12.07034631791048	$1.521E-9$	16.0

Using Richardson's extrapolation formulae:

$$\begin{aligned} I - T_n &\approx \frac{1}{3}\left[T_n - T_{\frac{1}{2}n}\right] \\ R_n &= T_n + \frac{1}{3}\left[T_n - T_{\frac{1}{2}n}\right] \end{aligned}$$

n	T_n	$I - T_n$	$\frac{1}{3}\left[T_n - T_{\frac{1}{2}n}\right]$	R_n	$I - R_n$	*Ratio*
2	-17.38925933	5.319				
4	-13.33602285	1.266	1.351	-11.98494401978457	$-8.540E-2$	
8	-12.38216243	$3.118E-1$	$3.179E-1$	-12.06420895721694	$-3.127E-3$	13.9
16	-12.14800410	$7.766E-2$	$7.805E-2$	-12.06995132327725	$-3.950E-4$	15.5
32	-12.08974212	$1.940E-2$	$1.942E-2$	-12.07032145605332	$-2.486E-5$	15.9
64	-12.07519410	$4.848E-3$	$4.849E-3$	-12.07034475993145	$-1.556E-6$	16.0
128	-12.07155818	$1.212E-3$	$1.212E-3$	-12.07034621906909	$-9.732E-8$	16.0
256	-12.07064928	$3.030E-4$	$3.030E-4$	-12.07034631030645	$-6.083E-9$	16.0