TRELLYS and Beyond: Type Systems for Advanced Functional Programming

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What is Functional Programming?

Central Ideas:

- Functions as data (higher-order functions, partial applications).
  - `List.map ((+) 10) [ 1 ; 2 ; 3 ]`
- Procedures behave like mathematical functions.
  - monads for making effectful procedures behave.

Culture:

- memory safety (via types)
- compile-time debugging via types
- type inference
- garbage collection
- pattern matching, inductive data
- module systems

The State of FP

Practical ↞— FP —↠ Verifiable

Software Transactional Memory
Lightweight threads
Continuing compiler improvements
...

GADTs, dependent types
Resource typing (HTT)
More powerful type inference
...

Sweet spot, or missing all targets?

This Talk

- Verifiability: TRELLYS
  - practical dependently typed language.
  - quasi-implicit products.
  - briefly: effect system for termination, termination casts.
- Practicality: BLAISE
  - goal: memory-safe functional programming without GC.
  - idea: divide pointers into primary and alias.
  - primary pointers used linearly.
  - alias pointers must have reciprocal backpointer.
  - *early-stage work.*

Design Goals and Design Ideas for TRELLYS

The TRELLYS Project

Goal: a new functional language with dependent types.

- Why Dependent Types Matter™:
  - incremental verification.
  - standard example:

```
[ 10 ; 20 ; 30 ] : vec int 3

append : Forall(A:type)(n1 n2 : nat).
         l1 : vec A n1 -> l2 : vec A n2 -> vec A n1+n2
```

  - small intellectual step from `list A` to `vec A n`.
  - other formal methods: much bigger leap!
- TRELLYS: combine resources to build compiler, libraries, etc.
- Involve broader community (working groups).
- First step: devise core language (PIs + SPJ, CM, BB, WS).

Goals for TRELLYS Core Language

- CBV, mutable state, inductive types, gen. recursion.
- Straightforward syntax-directed type checking.
  - surface language: type inference, automated reasoning.
  - elaboration to core adds annotations.
  - no fancy algorithms in the core.
  - *trustworthy* type checking for core.
- Logically sound fragment under Curry-Howard.
  - general recursion => unsound proof.
  - want a sound notion of proof.
  - without proofs, would need runtime checks.

Technical Goals for TRELLYS Core Language

- General recursive functions, sound proofs.
- Implicit products.
  - specificational ("ghost") data.

```
append : Forall(A:type)(n1 n2 : nat).
         l1 : vec A n1 -> l2 : vec A n2 -> vec A n1+n2
```

  - not computationally relevant.
  - dropped during compilation, and formal reasoning.
- `type:type`
  - most powerful form of polymorphism known.
  - allows type-level computation, data structures:

```
[ int ; bool ; int -> int] : list type
```

- Non-constructive reasoning (AS).
  - already distinguishing terminating/general recursive.
  - so distinguish proofs and programs.
  - motivation: case-split on termination of term.
  - cf. *Logic-Enriched Type Theory*, Zhaohui Luo.

Aside: Non-Constructive Reasoning

- Suppose quantifiers range over values.
- Define `foldr`:

```
(foldr f b [])     = b
(foldr f b (x:xs)) = (f x (foldr f b xs))
```

- Suppose *u* is an assumption of:

```
Forall(a:A)(b:B). (f a b) = (g a b)
```

- Then:

```
(foldr f b l) = (foldr g b l)
```

- Proof by induction on l. Step case:

```
(foldr f b (a:l')) = (f a (foldr f b l'))   // evaluation
                   = (f a (foldr g b l'))   // IH
                   = (g a (foldr g b l'))   // not allowed by u!
                   = (foldr g b (a:l'))
```

- Solution: case split on whether or not `(foldr g b l')` terminates.

Implicit Products

- Want erasure: $|append\ A\ n_1\ n_2\ l_1\ l_2| = append\ l_1\ l_2$.
- For compilation, and for formal reasoning.
- General theoretical approach:
  1. define unannotated system:
     - terms t, types T.
     - type assignment $\Gamma \vdash t : T$, reduction $t \leadsto t'$.
     - type assignment may not be algorithmic.
  2. do metatheory for unannotated system.
  3. define annotated system:
     - annotated terms a, types A.
     - algorithmic typing $\Gamma \Vdash a : A$.
     - erasure $|a| = t$, $|A| = T$.
  4. conclude metatheory for annotated system.

Example: $\mathtt{T}^{\mathtt{vec}}$

- Gödel's System T + equality types, vector types.
- Unannotated terms:

$$\begin{array}{rcl} t & ::= & x \mid (t\ t') \mid \lambda x.t \mid 0 \mid (S\ t) \mid (R_{\mathtt{nat}}\ t\ t'\ t'') \\ & & \mid \mathtt{nil} \mid (\mathtt{cons}\ t\ t') \mid (R_{\mathtt{vec}}\ t\ t'\ t'') \mid \mathtt{join} \end{array}$$

- Reduction relation $t \leadsto t'$ as expected.
- Unannotated types:

$$T\ ::=\ \mathtt{nat} \mid \langle \mathtt{vec}\ T\ t \rangle \mid \Pi x : T.T' \mid \forall x : T.T' \mid t = t'$$

- $\forall x : T.T'$ – implicit product.

Type Assigment: Implicit Products

- A form of intersection type.
- Studied for Implicit Calculus of Constructions [Miquel01].
- No term constructs in unannotated system.

$$\frac{\Gamma, x : T' \vdash t : T \quad x \notin FV(t)}{\Gamma \vdash t : \forall x : T'.T} \qquad \frac{\Gamma \vdash t : \forall x : T'.T \quad \Gamma \vdash t' : T'}{\Gamma \vdash t : [t'/x]T}$$

Type Assigment: Equality Proofs

- Do not rely on an algorithmic conversion relation.
- Use explicit casts to change types of terms.
- Casts are computationally irrelevant.
- So no term construct for unannotated system.

$$\frac{t \downarrow t' \quad \Gamma\ Ok}{\Gamma \vdash \texttt{join} : t = t'} \qquad \frac{\Gamma \vdash t : t_1 = t_2 \quad \Gamma \vdash t' : [t_1/x]T \quad x \notin dom(\Gamma)}{\Gamma \vdash t' : [t_2/x]T}$$

Annotated $\mathtt{T}^{\mathtt{vec}}$

$$a \;::=\; \ldots \mid (a\ a')^- \mid \lambda^- x : A.a \mid (\mathtt{join}\ a\ a') \mid (\mathtt{cast}\ x.A\ a\ a')$$

$$A \;::=\; \mathtt{nat} \mid \langle \mathtt{vec}\ A\ a \rangle \mid \Pi x : A.A' \mid \forall x : A.A' \mid a = a'$$

$$\begin{array}{lcl} |(a\ a')^-| & = & |a| \\ |\lambda^- x : A.a| & = & |a| \\ |(\mathtt{cast}\ x.A\ a\ a')| & = & |a'| \\ |(\mathtt{join}\ a\ a')| & = & \mathtt{join} \end{array}$$

$$\frac{\Gamma \Vdash a : A \quad \Gamma \Vdash a' : A' \quad |a| \downarrow |a'|}{\Gamma \Vdash (\mathtt{join}\ a\ a') : a = a'} \qquad \frac{\Gamma \Vdash a : a_1 = a_2 \quad \Gamma \Vdash a' : [a_1/x]A}{\Gamma \Vdash (\mathtt{cast}\ x.A\ a\ a') : [a_2/x]A}$$

Metatheory

- For unannotated system:
  - Standard results: type preservation, progress.
  - Strong normalization via reducibility argument.
- Lifting results to annotated system straightforward.

Large Eliminations

- Type-level computation.
- Many feel essential to dependent types.

$$T \quad ::= \quad \ldots \mid R\ t\ T\ (\alpha.T')$$

$$\frac{\Gamma \vdash t : R\ 0\ T\ (\alpha.T')}{\Gamma \vdash t : T} \qquad \frac{\Gamma \vdash t : R\ (S\ t')\ T\ (\alpha.T') \quad \Gamma \vdash t' : \mathtt{nat}}{\Gamma \vdash t : [R\ t'\ T\ (\alpha.T')/\alpha]T'}$$

Problem: Inconsistent Contexts

- Well-known issue if can equate types:

$$u : \texttt{nat} = (\Pi x : \texttt{nat}.\texttt{nat}) \;\vdash\; (0\ 0) : \texttt{nat}$$

- Can also type diverging terms.
- Same problem arises with large eliminations.
- Problem is even worse with implicit products:

$$\frac{u : \texttt{nat} = (\Pi x : \texttt{nat}.\texttt{nat}) \;\vdash\; (0\ 0) : \texttt{nat}}{\vdash (0\ 0) : \forall u : \texttt{nat} = (\Pi x : \texttt{nat}.\texttt{nat}).\ \texttt{nat}}$$

- Stuck terms would be typable in empty context!
- Solution: quasi-implicit products.

Quasi-Implicit Products

- Idea: do not completely erase implicit abstraction, application.
- Unannotated system:

$$t \quad ::= \quad \ldots \mid (\lambda.t) \mid (t)$$

$$\frac{\Gamma, x : T' \vdash t : T \quad x \notin FV(t)}{\Gamma \vdash (\lambda.t) : \forall x : T'.T} \quad \frac{\Gamma \vdash t : \forall x : T'.T \quad \Gamma \vdash t' : T'}{\Gamma \vdash (t) : [t'/x]T}$$

- Do not reduce beneath quasi-implicit abstraction.
- Meta-theory (including SN) preserved.
- See "Equality, Quasi-Implicit Products, and Large Eliminations" [pending]

Another Design Sketch: $\mathtt{T}^{\mathtt{eq}\downarrow}$

- "Termination Casts: A Flexible Approach to Termination with General Recursion" [PAR'10].
- Type-and-effect system for termination/possible divergence.
- Include equality types, termination types.
- Termination types reflect the termination effect.
- Terms and types:

$$\begin{array}{lcl} \theta & ::= & \downarrow \mid ? \\ A & ::= & \mathbf{nat} \mid \Pi^{\theta} x : A.A' \mid a = a' \mid \mathbf{Terminates}\ a \end{array}$$

$$\begin{array}{lcl} a & ::= & y \mid a\,a' \mid \lambda y : A\,.\,a \mid 0 \mid \mathbf{S}\,a \\ & \mid & \mathbf{rec_{nat}}\ g(y\ p) : A = a \mid \mathbf{rec}\ g(y : A) : A' = a \mid \mathbf{case}\ x.A\ a\ a'\ a'' \\ & \mid & \mathbf{join}\ a\,a' \mid \mathbf{cast}\ x\,.\,A\,a'\,a \mid \mathbf{terminates}\ a \mid \mathbf{reflect}\ a\,a' \mid \mathbf{inv}\ a\,a' \\ & \mid & \mathbf{contra}\ A\,a \mid \mathbf{abort}\ A \end{array}$$

Termination Casts

- Used to change the effect for a term.
- Proofs of termination first-class.
- External vs. internal verification:
  - Internal: type the function as total.

$$plus \; : \; \Pi x^{\downarrow} : \mathbf{nat}.\Pi y^{\downarrow} : \mathbf{nat}.\mathbf{nat} \; \downarrow$$

  - External: write a proof that the function is total.

$$\begin{array}{lcl} plus & : & \Pi x^{?} : \mathbf{nat}.\Pi y^{?} : \mathbf{nat}.\mathbf{nat} \; ? \\ plus\_tot & : & \Pi x^{\downarrow} : \mathbf{nat}.\Pi y^{\downarrow} : \mathbf{nat}.\mathbf{Terminates} \; (plus \; x \; y) \; \downarrow \end{array}$$

- $\mathtt{T}^{\mathtt{eq}\downarrow}$ supports both.

Selected Typing Rules ($\Gamma \Vdash a : A\,\theta$)

$$\frac{\Gamma \Vdash a : A\,\theta \qquad \Gamma \Vdash a' : \textbf{Terminates}\ a\ \downarrow}{\Gamma \Vdash \textbf{reflect}\, a\, a' : A\,\theta'}\ \text{AT\_REFLECT}$$

$$\frac{\Gamma \Vdash a : A\ \downarrow}{\Gamma \Vdash \textbf{terminates}\, a : \textbf{Terminates}\ a\,\theta}\ \text{AT\_REIFY}$$

$$\frac{\begin{array}{l} p \not\in \textbf{fv}\,|\,a\,| \\ \Gamma' \ = \ \Gamma\,,\, g\ :\ \Pi^{?}x\!:\!\textbf{nat}.A\,,\ y\ :\ \textbf{nat} \\ \Gamma'' \ = \ \Gamma'\,,\, p\ :\ \Pi^{\downarrow}x_1\!:\!\textbf{nat}.\Pi^{\downarrow}u\!:\!y\ =\ \textbf{S}\,x_1.\textbf{Terminates}\ (\,f\,x_1\,) \\ \Gamma'' \Vdash a : [\,y\,/\,x\,]\,A\ \downarrow \end{array}}{\Gamma \Vdash \textbf{rec}_{\textbf{nat}}\ g(y\ p) : A = a : \Pi^{\downarrow}x\!:\!\textbf{nat}.A\,\theta}\ \text{AT\_RECNAT}$$

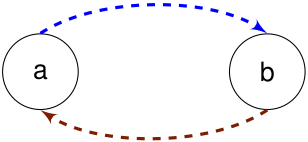
Taming Aliased Structures with Bidirectional Pointers: Blaise

Eliminating Garbage Collection

- GC pros: greater productivity, fewer bugs.
- GC cons: performance hit, unpredictability, complexity.
- Hard to use for real-time systems.
- Resource typing to the rescue?
  - statically track resources.
  - ensure no double deletes, no leaks.
  - alias types, L^3, HTT [Cornell/Harvard PL].
  - Stateful views [Xi et al.]
- Explicitly modeling locations is pretty heavy.
- Alternative: find and enforce safe abstractions for aliased structures.

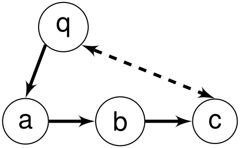
A Safe Abstraction: Bidirectional Pointers

- Idea: divide reference graph into primary/alias pointers.
  - primary pointers should form spanning trees.
  - alias pointers for all other edges.
- Each alias pointer must have a reciprocal backpointer.



- To delete a cell:
  - only delete via a primary pointer.
  - must first disconnect all alias pointers.
  - for disconnected alias pointer: must patch up reciprocal pointer.
- Overhead reasonable: one extra word per alias pointer.
- To enforce this abstraction: symbolic simulation.

Example: FIFO Queues



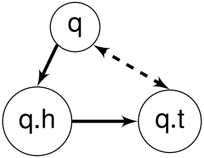

```
type node a
type queue a

inode : all a . Cell (d : a , n : node a) . node a
enode : all a . Cell (d : a , h : queue a) . node a

mk_queue : all a . Cell (h : node a , t : node a) . queue a
empty_queue : all a . Cell () . queue a
```

Symbolic Simulation by Example

```
fun insert<a>(consume d : a ,
              update q : queue a).
  (case q of
    empty_queue ->
      let e = new enode <a> in
      e.d = d;
      update q to mk_queue <a>;
      connect e.h q.t;
      q.h = e
  | mk_queue ->
*     let x = disconnect q.t in
      let e = new enode <a> in
      e.d = d;
      connect e.h q.t;
      let y = (take x.d) in
      update x to inode <a>;
      x.d = y;
      x.n = e); 0
```



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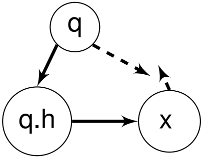

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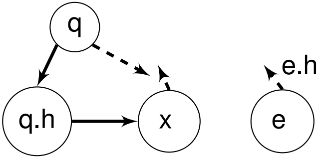

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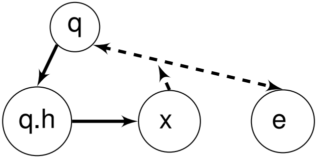

Symbolic Simulation by Example

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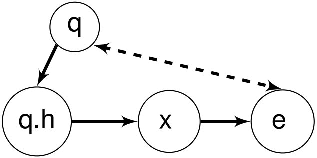

Conclusion

- FP and Verifiability: TRELLYS:
  - combine dependent types, general recursion.
  - quasi-implicit products.
  - termination casts.
  - *next step:* putting it all together, with `type:type`.
- FP and Practicality: BLAISE:
  - programming discipline: primary/alias pointers, reciprocal alias pointers.
  - enforce by symbolic simulation.
  - *next step*: define symbolic simulation, implement.
- For more info:
  - "Equality, Quasi-Implicit Products, and Large Eliminations"
  - "Termination Casts: A Flexible Approach to Termination with General Recursion"
  - See also QA9 (blog).

Thanks for having me!