22C:44 Homework 6 Solution

1. The result of linear probing is:

Table	Probes
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 10	1
22, 0, 0, 0, 0, 0, 0, 0, 0, 0, 10	1
22, 0, 0, 0, 0, 0, 0, 0, 0, 31, 10	1
22, 0, 0, 0, 4, 0, 0, 0, 0, 31, 10	1
22, 0, 0, 0, 4, 15, 0, 0, 0, 31, 10	2
22, 0, 0, 0, 4, 15, 28, 0, 0, 31, 10	1
22, 0, 0, 0, 4, 15, 28, 17, 0, 31, 10	2
22, 88, 0, 0, 4, 15, 28, 17, 0, 31, 10	2
22, 88, 0, 0, 4, 15, 28, 17, 59, 31, 10	5

The total number of probes is 16. The zeros in this and the next two tables should be read as `nil`.

The result of quadratic probing is:

Table	Probes
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 10	1
22, 0, 0, 0, 0, 0, 0, 0, 0, 0, 10	1
22, 0, 0, 0, 0, 0, 0, 0, 0, 31, 10	1
22, 0, 0, 0, 4, 0, 0, 0, 0, 31, 10	1
22, 0, 0, 0, 4, 0, 0, 0, 15, 31, 10	2
22, 0, 0, 0, 4, 0, 28, 0, 15, 31, 10	1
22, 0, 0, 17, 4, 0, 28, 0, 15, 31, 10	4
22, 0, 88, 17, 4, 0, 28, 0, 15, 31, 10	9
22, 0, 88, 17, 4, 0, 28, 59, 15, 31, 10	3

The total number of probes is 23.

The result of double hashing is:

Table	Probes
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 10	1
22, 0, 0, 0, 0, 0, 0, 0, 0, 0, 10	1
22, 0, 0, 0, 0, 0, 0, 0, 0, 31, 10	1
22, 0, 0, 0, 4, 0, 0, 0, 0, 31, 10	1
22, 0, 0, 0, 4, 15, 0, 0, 0, 31, 10	3
22, 0, 0, 0, 4, 15, 28, 0, 0, 31, 10	1
22, 0, 0, 17, 4, 15, 28, 0, 0, 31, 10	2
22, 0, 0, 17, 4, 15, 28, 88, 0, 31, 10	3
22, 0, 59, 17, 4, 15, 28, 88, 0, 31, 10	3

The total number of probes is 16.

You were only required to write down the final table and the total number of probes for each hashing technique.

In a separate file you will find *Mathematica* code that produces the above answers.

2. In the tables below, "Minimum" denotes the fewest number of elements hashed to a slot, "Maximum" denotes the largest number of elements that hash to a slot, and "Empty Slots" denotes the number of slots to which no element hashes. For $A = (\sqrt{5} - 1)/2$ we obtain the following table

N	Minimum	Maximum	Empty Slots
1000	9	11	0
10000	99	102	0
100000	998	1002	0

For $A = 7/11$ the table is:

N	Minimum	Maximum	Empty Slots
1000	0	91	89
10000	0	910	89
100000	0	9091	89

For $A = 31/127$ the table is:

N	Minimum	Maximum	Empty Slots
1000	7	16	0
10000	78	158	0
100000	787	1576	0

The *Mathematica* program that produced these numbers is in a separate file.

3.
   - It is more likely that the last two parking slots are next to each other.
   - This question is an illustration of hashing using open-addressing with linear probing. Entry of cars into the circular road corresponds to keys hashing into slots; cars occupying the first available slot corresponds to resolving collisions using open-addressing with linear probing.

     With linear probing long runs of occupied slots are likely. This is why two adjacent empty slots are more likely that two empty slots on opposite sides of the circular road.

4.
   - If we simply write `nil` into a slot from which we have deleted an element, then in searching for a key k we will not be able to distinguish between the following two situations when we encounter a `nil` in a slot: (i) The slot has a `nil` because k does not exist in the table or (ii) The slot has a `nil` because the element in that slot was deleted (even though k is in the table).
   - The pseudocode for `Hash-Search` given in the book on page 234 can be used unchanged. The pseudocode for `Hash-Insert` needs one small change: one line (3) replace "**if** $T[j] =$ `NIL`" by "**if** $T[j] =$ `NIL` or $T[j] = \perp$." The pseudocode for `Delete` is as follows:

```
Hash-Delete(T, k) {
        i ← Hash-Search(T, k);
        if (i != nil) then T[i] ← ⊥
}
```

   - One way to clean the table of all the $\perp$ symbols would be to reinsert all the elements in the table and then change all the remaining $\perp$ symbols to `NIL`.