

# Homework 4

**22C:44 Algorithms, Fall semester 2000**

Four problems, ten points each. Due in class on Thursday, Sept. 28.

- 1 Design algorithm **Delete** for deleting an arbitrary element of a heap. **Delete**( $A[1 \dots n]$ ,  $i$ ) should remove element  $A[i]$  from heap  $A$  and rearrange the remaining  $n - 1$  elements to maintain the heap property. Your algorithm should run in  $O(\log n)$  worst-case time. You may use any algorithms presented in the class as subroutines.
- 2 Exercises 8.1-1 (page 155) and 8.3-2 (page 163) of the text-book.
- 3 Design the modified **Partition** we used in the class during the analysis of the randomized quicksort. **Partition**( $A, p, r$ ) should (a) use the first element  $A[p]$  as the pivot, (b) compare the pivot and all other elements  $A[p + 1 \dots r]$  exactly once, (c) perform no other comparisons between the elements, (d) re-order the elements in such a way that the pivot moves to position  $q$ , for some  $p \leq q \leq r$ , all elements smaller than the pivot move before position  $q$  and all elements larger than the pivot move after position  $q$ , (e) return number  $q$ , the new position of the pivot, and (f) run in the linear  $\Theta(r - p)$  time.
- 4 Let us investigate the problem of sorting arrays  $A[1 \dots n]$  that are known to be almost ordered initially in the sense that only some elements close to each other may be in the wrong order. More precisely, there exists a constant  $c$  such that whenever two element  $A[i]$  and  $A[j]$  are in the wrong order then  $|j - i| \leq c$ .
  - (a) In this case, what is the worst-case time complexity of the **BetterBubbleSort** algorithm of Homework assignment # 1. Justify (=prove) your answer.
  - (b) Design a linear time algorithm based on quicksort. Analyze the complexity of your algorithm to prove that it indeed runs in the  $\Theta(n)$  worst-case time. (Hint: modify the partitioning program to run in constant time.)