Homework 3
22C:196 Computational Combinatorics
Due on November 13, 2001

Part I

For the following 3 problems, you do *not* have to write code.

1. Let $p = (p_1, p_2, \ldots, p_n)$ be an n-permutation. Starting with an empty tableau, insert the elements $p_1, p_2, \ldots, p_n$ in that order and construct a tableau called P. What is the connection between the length of a longest increasing sequence in p and the shape of P? Similarly, what is the connection between the length of a longest decreasing sequence in p and the shape of P? Prove your assertions. Using this connection deduce the Erdös-Szekeres theorem: any sequence of n^2 distinct numbers has an increasing subsequence or a decreasing subsequence of size at least n.

2. Let t_n be the number of involutions of length n. Prove that $e^{z+z^2/2}$ is the exponential generating function of $(t_0, t_1, t_2, \ldots)$.

3. Use the Prüfer correspondence to show that the number of labeled trees with exactly t leaves is
$$\frac{n!}{t!} \cdot \left\{ {n-2 \atop n-t} \right\}.$$

Part II

For the following 2 problems you have to write *Mathematica* code or perform experiments with *Combinatorica* functions. Submit a *Mathematica* notebook containing solutions to these three problems.

1. Implement Savage's algorithm for generating integer partitions in Gray code order. The complete algorithm is described in Savage's paper "Gray Code Sequence of Partitions," *Journal of Algorithms*, 10, 577-595, 1989.

2. Recall that Ehrlich proved the impossibility of generating set partitions in "strict" Gray code order. Set partitions are in strict Gray code order if for any pair of consecutive set partitions their RG functions differ in exactly one position and by 1 in that position. However, Ehrlich presented an algorithm for generating set partitions in "relaxed" Gray code order. In this order, consecutive RG functions p and q differ in exactly one position and in that by 1, unless (i) the position contains the largest element in the RG function in which case it can become 1 or (ii) the position contains 1 in which case it can become the largest element m or $m + 1$. Implement Ehrlich's algorithm for generating set partitions in "relaxed" Gray code order. This algorithm was not covered in class and so look at Page 508 of Ehrlich's paper "Loopless Algorithms for Generating Permutations, Combinations, and Other Combinatorial Configurations," *JACM*, 20(3), 500-513, 1979.
