

Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 1

The homework is due in class on Thursday, February 5th. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom.

1. Let $\text{Even} : \{0,1\}^* \rightarrow \{0,1\}$ be the function defined as follows: $\text{Even}(x) = 1$ if $|x|$ is even, and 0 otherwise. Note that $|x|$ denotes the length of x . For example, $\text{Even}(0110) = 1$, and $\text{Even}(000) = 0$. Describe a Turing Machine that computes the function Even . What is its running time, in big-Oh notation, as a function of the input size? (3 points)
2. Let $f : \{0,1\}^* \rightarrow \{0,1\}$ be defined as follows: $f(x) = 1$ if $x = 0^n 1^n$ for some integer $n \geq 0$, and $f(x) = 0$ otherwise. For example, $f(0011) = 1$, $f(011) = 0$, and $f(1100) = 0$. Describe a Turing Machine that computes the function f . What is its running time, in big-Oh notation, as a function of the input size? (3 points)
3. Let Sum be a function that takes as input a string of the form $x\#y$, where $x, y \in \{0,1\}^*$. On such inputs, we treat x as y as integers represented in binary; Sum evaluates to the binary representation of their sum. For example, $\text{Sum}(11\#10) = 101$, and $\text{Sum}(001\#1111) = 10000$. Describe a Turing Machine that computes the function Sum . What is its running time, in big-Oh notation, as a function of the input size? (4 points)

For this homework, we will expect a full TM description for the first two problems. For the third problem, a full TM description is fine. You can also skip the details of the phases that may involve copying and reversing, but be very clear about how these phases interface with the actual adding phase. For each problem, you may use as many alphabet symbols or tapes as convenient. Please complement the description of the transition function by high level comments; this will aid the reader's understanding.