22C : 031 (CS : 3330 : 0001) Algorithms

Homework 4

March 21, 2012

This homework is based on our discussions of algorithm design using recursion and divide-and-conquer. The homework is worth 10 points.

1. In an array $A[1..n]$ of integers, a pair of numbers $A[i]$ and $A[j]$ form a *significant inversion* if $i < j$ and $A[i] > 3A[j]$. By modifying our algorithm for counting inversions, give an $O(n \log n)$ algorithm for counting the number of significant inversions in a given array. (3 points)

2. We are given an array $A[1..n]$ of integers with the special property that $A[1] \geq A[2]$ and $A[n-1] \leq A[n]$. We say that an element $A[x]$ is a *local minimum* if it is less than or equal to both its neighbors, that is, $A[x-1] \geq A[x]$ and $A[x] \leq A[x+1]$. For example, there are six local minima in the example array in Figure 1. We can obviously find a local minimum in $O(n)$ time by scanning through the array. Describe an $O(\log n)$ time algorithm for finding one local minimum. **Hint:** With the given boundary conditions, the array must have at least one local minimum. Why? (3.5 points)

3. You are at a political convention with n delegates, each one a member of exactly one political party. There are multiple parties. It is impossible to tell which political party any delegate belongs to; in particular, you will be summarily ejected if you ask. However, you can determine whether any two delegates belong to the *same* party or not by introducing them to each other – members of the same party always greet each other with smiles and friendly handshakes; members of different parties always greet each other with angry stares and insults.

   Suppose that a majority (more than half) of the delegates are from the same political party. Describe an algorithm that identifies a member (any member) of the majority party using only $O(n \log n)$ introductions. (3.5 points)

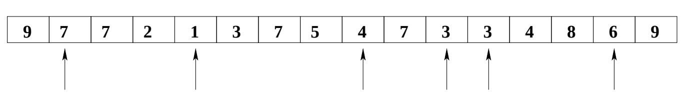

Figure 1: Local Minima in an Array

The homework is due Monday, March 5, in class; if you can't make it to class on that day, just make sure you get it to me by that time.