Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 4

The homework is due in class on Tuesday, November 10th. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom. Each question is worth 2.5 points.

1. Recall the language

$$\texttt{INDSET} = \{\langle G, k\rangle \mid G \text{ has an independent set of size } k\}.$$

   Suppose that $\texttt{INDSET} \in P$. Show that there is a polynomial time algorithm that, given a graph G and a number k such that G does have an indpendent set of size k, outputs an independent set of size k in G.

   Note that we are asking you to show that the search problem is solvable in polynomial time given that the corresponding decision problem is. You need to do this directly, without recourse to Theorem 2.18 in Section 2.5.

2. We saw that coNP could be defined in two ways:

   (a) A language L belongs to coNP if its complement $\overline{L}$ belongs to NP.

   (b) A language L belongs to coNP if there exists a polynomial $p : \mathbb{N} \to \mathbb{N}$ and a polynomial time TM M such that for every $x \in \{0,1\}^*$,

$$x \in L \Leftrightarrow \forall u \in \{0,1\}^* \text{ such that } |u| \leq p(|x|), M(x,u) = 1.$$

   Show that the two definitions are equivalent, that is, they define the same class of languages. (This is basically Exercise 2.24.)

3. Show that NP = coNP if and only if `3SAT` and `TAUTOLOGY` are polynomial-time reducible to each other. (If you prefer, you can replace `TAUTOLOGY` by $\overline{\texttt{3SAT}}$ in the problem statement.) (This is Exercise 2.26. Note that `3SAT` is the same as what we have been calling `3CNF-SAT`.)

4. Show that there is a language in SPACE(n^6) that is not in SPACE(n^2). This is a special case of the Space Hierarchy Theorem, which is Theorem 4.8 in the text. Model your proof on that of the Time Hierarchy Theorem (Theorem 3.1). See ICON for our notes on the latter.