22C : 231 Design and Analysis of Algorithms
Homework 5
Due Tuesday, December 2

You are expected to do the homework assignments on your own without consulting human and non-human sources (like web pages or books) for the solutions.

For doing reductions in the following questions, start from one of these: 3CNF-SAT, vertex cover, independent set, set cover, 3-coloring, 3-dimensional matching, and subset sum.

1. Our company has a set of employees X, and we have identified teams $S_1, \ldots, S_m$ of employees; each team will work together on a particular project in the next year. An employee can belong to more than one team. It turns out that all these projects require two new skills A and B which none of the employees have. We would like to train our employees so that they can acquire these skills. Since acquiring new skills is a time-consuming process, we can afford to train each of our employees on only one skill. We would like to ensure that each team has an employee trained in skill A and an employee trained in skill B.

   Thus the following algorithmic question: given a ground set X, and subsets $S_1, S_2, \ldots, S_m$ of X, is it possible to partition X into two subsets X_A and X_B so that for each $1 \leq i \leq m$, $S_i \cap X_A \neq \emptyset$ and $S_i \cap X_B \neq \emptyset$?

   Show that this problem is NP-complete.

2. Given a set of graphs $G_1 = (V, E_1)$, $G_2 = (V, E_2), \ldots, G_m = (V, E_m)$ all on the same vertex set V, and an integer k, where $1 \leq k \leq m$, determine if it is possible to combine the edge sets of k of these graphs to obtain a graph on vertex set V that is *connected*.

   Show that this problem is NP-complete.

3. Suppose V is a set of elements and we are given an integer distance $d(i, j) > 0$ between every two elements i and j in V. Given a subset $U \subseteq V$, we define its *extent* as the *maximum* distance between any pair of elements in U. The extent of a one-element set is defined to be 0. The algorithmic problem is: given such a V, the pairwise distances, and an integer k, is it possible to partition V into at most 3 subsets so that the sum of the extents of the 3 subsets is at most k ?

   Show that this problem is NP-complete.